Chemistry 20262 | Spring 2018
Professor J. Daniel Gezelter | Due Fri. 1/26/2018

Problem Set 1 (Complex Numbers, Euler's Relation)

1. If z^* is the complex conjugate of z, show that $\mathrm{Re}(z) = (z+z^*)/2$ and that $\mathrm{Im}(z) = (z-z^*)/2i$.
2. Evaluate the following:
   (a) $\mathrm{Re}\left[\frac{3+i}{1-i}\right]$
   (b) $\left|\frac{(1-i)^3}{(3+2i)^2}\right|$
   (c) $\sum_{k=0}^{5} i^k$
   (d) $\mathrm{Im}\left[\frac{1}{3+i} - \frac{1}{3-i}\right]$
3. Consider the following functions of complex numbers $z = x+iy$. Rewrite each of these functions in the form of real-valued functions: $w = u(x,y) + iv(x,y)$:
   (a) z^3
   (b) $\frac{1}{(1-z)^2}$
   (c) $\frac{z^*}{z}$
4. Express the following numbers in the form $re^{i\theta}$, where r and θ are real numbers:
   (a) $6i$
   (b) $4-\sqrt{2}i$
   (c) $-1-2i$
   (d) $1+i$
5. Express the following numbers in the form $x+iy$, where x and y are real numbers:
   (a) $e^{i\pi/4}$
   (b) $6e^{2\pi i/3}$
   (c) $e^{(\pi/4)i+\ln 2}$
   (d) $e^{-2\pi i} + e^{4\pi i}$
6. Show that $\cos\theta = (e^{i\theta} + e^{-i\theta})/2$ and that $\sin\theta = (e^{i\theta} - e^{-i\theta})/2i$.
7. Evaluate the following into a simpler form:
   (a) $(1+i)^{10}$
   (b) $(1-i)^{12}$

8. Use Euler's formula to evaluate

   a) i^i

   b) $\sqrt{i}$

9. Use Euler's formula to show that

$$\int_0^{2\pi} \sin(nx)\sin(mx)dx = \begin{cases} 0 & n \neq m \\ \pi & n = m \end{cases}$$

10. Do any additional homework problems from the Mathematica Practical Lab #1.

11. Extra Credit

    The number $\sqrt{2}$ is irrational:

$$\sqrt{2} = 1.41421356...\text{(decimal expansion neither truncates nor repeats)}$$

There are rational numbers close (arbitrarily close) to $\sqrt{2}$. For example:

$$\begin{aligned} \frac{7}{5} &= 1.4 \\ \frac{17}{12} &= 1.41\overline{6} \\ \frac{41}{29} &= 1.\overline{4137931034482758620689655172} \end{aligned}$$

Find a rational number p/q that is closer to $\sqrt{2}$ than the above examples where q is *not divisible by 10*. Just report p and q; a decimal expansion is not necessary.

There's some wonderful history to this problem - you might want to do some background reading on continued fractions, the Babylonian method, and Mediants of fractions.

The student who gets the closest to $\sqrt{2}$ will win a cookie.