Chemistry 20262 — Spring 2018
Professor J. Daniel Gezelter — Due Fri. 4/20/2018

Problem Set 9
Probability

1. Show that $\sigma_f^2 \equiv \left\langle (f - \langle f \rangle)^2 \right\rangle = \left\langle f^2 \right\rangle - \langle f \rangle^2$ for an arbitrary quantity f.

2. Consider the Gaussian probability density $P(x)$ for the continuous variable x,

$$P(x) = Ae^{-a(x-x_0)^2}$$

   Determine the normalization constant A.

3. Using $P(x)$ from problem 2, compute $\langle x \rangle$, $\left\langle x^2 \right\rangle$, and σ_x

4. Do problem 21-2 in McQuarrie.

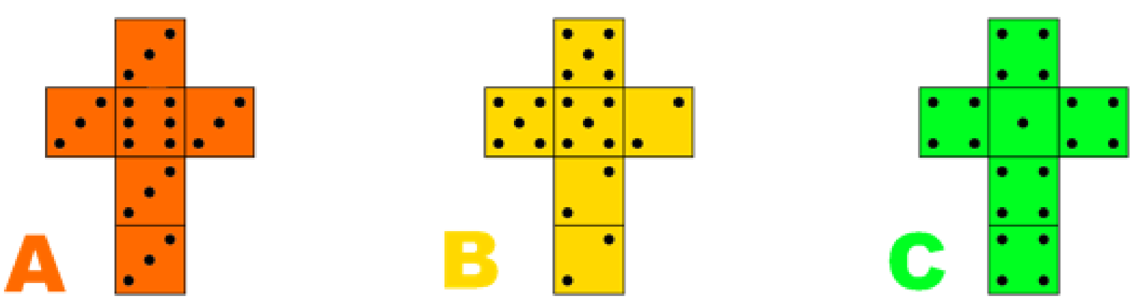
5. Do problem 21-6 in McQuarrie.

6. Do problem 21-14 in McQuarrie.

7. Suppose we have a set of non-standard dice that have a strange collection of numbers on their faces. That is:

Normal:	1	2	3	4	5	6
Die A:	3	3	3	3	3	6
Die B:	2	2	2	5	5	5
Die C:	1	4	4	4	4	4

   If you want a picture of what these dice look like and want to construct your own out of paper, you can cut out the dice shown below:



   Let's play a game in which each player picks a die and the two players then roll their respective dice at the same time. Whoever gets the highest value wins.

(a) Start by showing that dice A, B, and C all have the same expectation values (e.g. averages) as the normal dice.

(b) Show by tabulating all 36 possible combinations for the A-B contest and all 36 combinations for the B-C contest, that on average, die A beats die B (it will win against B more often than it will lose) with a probability of $\frac{7}{12}$, and that die B beats die C also with a probability of $\frac{7}{12}$. So it would appear that A is the strong die and C is the weak die. If this is truly the case, then these dice are *transitive*.

(c) Tabulate all 36 combinations for the A-C contest and show that C beats A with probability $\frac{25}{36}$. What other game has a non-transitive outcome?

(d) 2 points extra credit: What are the win/lose probabilities if each player has two identical dice?

8. Extra credit: Compute the quantity $\left\langle (x-\langle x\rangle)^4 \right\rangle$ for both the normalized Gaussian distribution (Problem 2) and the normalized Exponential distribution (Problem 6). Use this quantity to evaluate the *non-Gaussian parameter* for both distributions:

$$\alpha_2 = \frac{\left\langle (x-\langle x\rangle)^4 \right\rangle}{5\left\langle (x-\langle x\rangle)^2 \right\rangle^2} - \frac{3}{5}$$