Chemistry 20262 | Spring 2018
Professor J. Daniel Gezelter | Due Fri. 2/23/2018

Problem Set 4
(Critical Points, Integration, Lagrange Multipliers, DiffEQs)

1. Find the critical points and determine whether the function $f(x,y) = x^3 - 4x^2 - xy - y^2$ has a maximum, a minimum, or a saddle point at them.

2. Determine the volume under the surface described by $z = 1 - x^2 - y^2$ and over the square with vertices $(\pm 1, 0)$ and $(0, \pm 1)$ in the $z = 0$ plane.

3. Show that $\int_0^1 dy \int_y^1 dx\, y\, e^{x^3} = \frac{1}{6}(e-1)$. Hint: Reverse the order of integration.

4. The fencing problem: You have a fixed length (ℓ) of fence material. Find the rectangle of perimeter ℓ that has the shortest diagonal.

5. The wrapping paper problem: You have a fixed area (A) of wrapping paper available. Find the maximum volume of a right cylinder that you can wrap with total surface area A (including top and bottom).

6. Find the general solutions of:
   (a) $\frac{dy}{dx} = x^2 - 3x^2 y$
   (b) $\frac{dy}{dx} + \frac{2}{x} y = x^2 + 2$
   (c) $t \frac{ds}{dt} = (3t+1)s + t^3 e^{3t}$
   (d) $(x + y^2)\frac{dy}{dx} = 1$

7. Extra Credit: You are on a rescue crew trying to help a hiker who has gotten stuck in the mountains get back to town (which is at the bottom of the lowest valley on the terrain). The hiker has a two-way radio, but he is blind and has feet that are stuck pointing along the x- and y- axes of your world. He cannot see the terrain, but his feet are very sensitive and can measure the slope in each of the x and y directions (the hiker has partial derivative feet). Give the hiker a set of instructions that will help him get back to town.

   Will your set of instructions work in all possible terrains? Under what conditions will your instructions fail? Do you think it is possible to find a set of instructions that will work in all possible terrains?