Chemistry 60649 Fall 2008
Professor J. Daniel Gezelter Due Mon. 11/1/2008

Problem Set 5

These problems will concentrate primarily on Angular momentum and the Hydrogen atom.

1. Consider the bending vibrations of CO_2, which can be roughly modeled as a 2-dimensional harmonic oscillator with the following Hamiltonian,

$$H = -\frac{\hbar^2}{2\mu}\frac{d^2}{dx^2} - \frac{\hbar^2}{2\mu}\frac{d^2}{dy^2} + \frac{k}{2}(x^2 + y^2). \tag{1}$$

   There are 2 ways to solve this problem. We'll do both.

   a) Assume that the wave function is a product of a function in x, and another function in y. Use the separation of variables method and your skill with Harmonic oscillator operators to find the energies and eigenstates of this system. Don't reinvent the wheel. Once you show that this is a separable problem, you can quote previous work on the Harmonic oscillator for energies and eigenstates.

   b) Convert the Hamiltonian to polar coordinates and show that the eigenfunctions may be then written as

$$\psi(r, \theta) = C_m R_m(r) e^{im\theta} \tag{2}$$

   where $R_m(r)$ satisfies the radial equation

$$\left[-\hbar^2\left(r^2\frac{\partial^2}{\partial r^2} + r\frac{\partial}{\partial r}\right) + m^2\hbar^2 + 2\mu r^2[V(r) - E]\right] R_m(r) = 0 \tag{3}$$

2. Problem 9.23 in Liboff.

3. Problem 9.26 in Liboff.

4. Extra Credit (Not required, but it is a good character-building exercise): First, go read problem 9.14 in Liboff, and make sure you understand everything there. Then do problem 9.18.