Chemistry 322 | Spring 2004
Professor J. Daniel Gezelter | Due Fri. 1/23/2004

Problem Set 1

1. Your physical chemistry class has twenty-three students. What is the probability that at least two classmates have the same birthday?

2. According to the kinetic theory of gases, the energies of molecules moving along the x-direction are given by $\epsilon_x = mv_x^2/2$ where m is the mass of the molecule and v_x is the velocity in the x-direction. The distribution of particles with a given velocity is given by the Boltzmann law,
$$p(v_x) = e^{-mv_x^2/2k_BT}$$
(This is sometimes called the Maxwell-Boltzmann distribution). Given that velocities can range from $-\infty$ to ∞,

   a) Write the probability distribution $p(v_x)$ so that it is correctly normalized,
   b) Compute the average energy, $\langle mv_x^2/2\rangle$,
   c) Find the average velocity $\langle v_x\rangle$, and
   d) Find the average momentum $\langle mv_x\rangle$.

3. A biological membrane contains N ion-channel proteins. The fraction of time that any one protein is open to allow ions to flow through is q. Express the probability $P(m, N)$ that m of the channels will be open at any given time.

4. The Monty Hall dilemma: You are a contestant on a game show. There are three closed doors: one hides a car, and two hide goats. (With each door having 1/3 probability of hiding the car.) You point to one door as your initial choice. The game show host, knowing what's behind each door, now opens one of the other doors to show you a goat. To win a car, you now get to make your final choice: should you stick with your initial choice, should you switch your choice, or does it matter at all if you switch?

5. Do problem 16-6 in McQuarrie & Simon.

6. Do problem 16-40 in McQuarrie & Simon.

7. 20 points extra credit: The Central Limit Theorem

   a) Write a simple computer program (in whatever language you'd like) which generates the sum
$$X_N = \sum_{i=1}^{N} \frac{x_i}{\sqrt{N}}$$
   where $\{x_1, ..., x_i, ...x_N\}$ are independent random numbers which are uniformly distributed on the interval $-1/2 < x_i < 1/2$. Your program should compute X_N at

least one million times and then construct a histogram of the X_N values you observe.

b) What are the maximum and minimum possible values for X_N?

c) Show (numerically) that for large N the distribution of X_N values looks Gaussian.

d) Where does the Gaussian approximation work best? Where does it fail?